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Time-Reversal Symmetry Violation and the Oscillating Universe†

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Abstract

The expressions for the fractional number of K^0's and $\bar{K}^0$'s in a neutral kaon beam are discussed with reference to time-reversal asymmetry. The suggested relation between the sign of Re ϵ (ϵ is the Lee-Wu T-violation parameter) and the cosmological arrow of time if CPT is broken is further clarified.

1. *Introduction*

In a previous article (Ne'eman, 1970), the experimentally established violation of CP symmetry in the decay of the long-lived K meson (Christenson *et al.*, 1964)—and the further possibility of CPT violation—were studied in the context of time-symmetric oscillating models of the universe. It was shown that the current assumption, according to which the contracting phase of the oscillation is reinterpreted as a time-inverted expansion, cannot be retained at all if CPT is violated; if only CP is violated, the assumption is allowed and involves inverting the definitions of matter and antimatter.

To describe the evolution of a K^0–$\bar{K}^0$ complex, the Lee–Oehme–Yang formula (Lee *et al.*, 1957) was used. This formula predicts the number of neutral K mesons remaining in the beam at any time t. In the present article we refine the argument and apply it to a different set of formulae which emphasize the observables involved.

In Section 2, the formulae for the fractional number of K^0's and $\bar{K}^0$'s in a neutral kaon beam are discussed, in an ordinary and in a time-inverted coordinate schemes. In Section 3 these formulae are used for defining a relation between the cosmological arrow of time and the behaviour of the microscopic K^0–$\bar{K}^0$ system.

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2. *Time Reversal Asymmetry in K^0–$\bar{K}^0$ Distinguishing Formulae*

We first consider at $t=0$ a beam of pure K^0's (strangeness $=+1$). For $t>0$, these particles will decay via the weak Hamiltonian eigen-states K_S and K_L, thus forming at time t—aside from the decay products—a number of $\bar{K}^0$'s. Using the parametrization of Lee and Wu (Lee *et al.*, 1967), with δ the *CPT* non-invariance parameter ($\delta=0$ if *CPT* is conserved) and ϵ the T non-invariance parameter, a direct calculation gives for the above beam (Aharony, 1970)

$$R^{K^0}(K^0,t)=\tfrac{1}{4}\{(1-4\,\mathrm{Re}\,\delta)\exp(-\gamma_L t)+(1+4\,\mathrm{Re}\,\delta)\exp(-\gamma_S t)+ + [(1+4i\,\mathrm{Im}\,\delta)\exp(i\Delta mt)+\text{c.c.}]\exp[-\tfrac{1}{2}(\gamma_L+\gamma_S)t]\} \quad (2.1)$$

$$R^{K^0}(\bar{K}^0,t)=\tfrac{1}{4}(1-4\,\mathrm{Re}\,\epsilon)\{\exp(-\gamma_S t)+\exp(-\gamma_L t)-2\cos\Delta mt\times \times\exp[-\tfrac{1}{2}(\gamma_L+\gamma_S)t]\} \quad (2.2)$$

$R^{K^0}(K^0,t)$ and $R^{K^0}(\bar{K}^0,t)$ are, respectively, the fractions of K^0 and of $\bar{K}^0$ particles in the beam at the kaon's proper time t. γ_L and γ_S are the inverse lifetimes of K_L and of K_S, and Δm is their mass difference. For a beam initially made of pure $\bar{K}^0$'s, $R^{\bar{K}^0}(\bar{K}^0,t)$ and $R^{\bar{K}^0}(K^0,t)$ will be given by the same formulae, except for a change in the signs of ϵ and of δ. (All expressions are to first order in ϵ and δ.)

We now wish to consider the same beam in a time reversed coordinate system. The expressions for reversed time, $-t$, are obtained from those for t by applying the time-reversal operation T. As shown in the previous article (Ne'eman, 1970) and by Zweig (1967), the equation of motion

$$i\frac{d}{dt}\psi=(M-i\Gamma)\psi \quad (2.3)$$

for the two-dimensional state-vector ψ describing the K^0–$\bar{K}^0$ complex (M and Γ are the 2×2 mass and decay matrices) is transformed under T to

$$i\frac{d}{dt'}\psi_T{}^*=(M^*-i\Gamma^*)\psi_T{}^* \quad (2.4)$$

where $t'=-t$. Therefore, $\psi_T{}^*(t')$ will exhibit a time evolution similar to that of $\psi(t)$, except for the transformation

$$\epsilon\to-\epsilon,\qquad \delta\to\delta \quad (2.5)$$

Since equation (2.1) describes the fractional number of K^0's in a beam beginning at $t=0$ with pure K^0, we can deduce that the fractional number of K^0's in the time-reversed coordinate system, described by $P^{K^0}(K^0,t')$, will have the same time dependence [note that equation (2.1) involves only δ, which does not change under T]:

$$P^{K^0}(K^0,-t)=R^{K^0}(K^0,t) \quad (2.6)$$

Similarly, we obtain from equations (2.2) and (2.5)

$$P^{K^0}(\bar{K}^0, -t) = (1 + 8\,\mathrm{Re}\,\epsilon)\, R^{K^0}(\bar{K}^0, t) \tag{2.7}$$

3. *A Relation Between Microscopic and Cosmological Arrows of Time*

In a time-symmetric oscillatory model, we would expect every physical situation to repeat itself after a time τ, τ being the oscillation period of the universe. Thus, the fractional number of K^0 and of $\bar{K}^0$ particles in the universe at the points A and B of maximum contraction (Fig. 1) should coincide. We can in fact restrict ourselves to a given volume element and discuss a subsector of the universe containing initially (at A) a beam of pure K^0 particles.

At times $t > 0$, the beam will decay, leading to a decrease in the number of K^0's and an increase in the number of $\bar{K}^0$'s, described by equations (2.1)–(2.2). These formulae give the time-evolution only for short times,

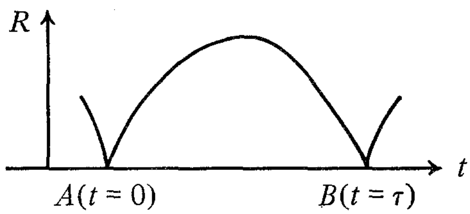

Figure 1

since they are based on the Wigner-Weisskopf approximation (Lee *et al.*, 1957). Still, if the beam exists at times larger than $\tau/2$, then we must assume that the decay products are contracted to reproduce the initial K^0 beam, since we demand complete identity of the physical states at A and at B.

By the time symmetry of the oscillating model, we may assume that the behaviour of the beam at the time $(\tau - t)$ approaching the point B is the same as at the time $-t$, given by eqs. (2.6)–(2.7). The left-hand sides of these equations were derived as the results of a decay process in the time-inverted contracting universe, but they may be reinterpreted as the fractional numbers of K^0's and $\bar{K}^0$'s needed at the time $(\tau - t)$ in order that the beam will end at the time τ as pure K^0.

Equation (2.6) thus represents a complete symmetry of the fractional number of K^0's in the beam with respect to the time $\tau/2$. Note that this symmetry is independent of *CPT* invariance, since it holds for any value of δ. There is no such symmetry in equation (2.7); the fractional number of $\bar{K}^0$'s in a beam of K^0's will reveal a symmetry with respect to $\tau/2$ only if $\mathrm{Re}\,\epsilon = 0$, hence if T is conserved!

If T is not conserved for the K^0–$\bar{K}^0$ system, as it now seems to be established (Casella, 1968, 1969; Achiman, 1969), we may use (2.7) to define the direction of the arrow of time. There are several possible ways to determine Re ϵ experimentally, e.g. by measuring the charge asymmetry in K_L leptonic decay (Schwartz *et al.*, 1967) or by other experiments measuring the overlap of the states K_L and K_S. In most of these experiments one has expressions with combinations of ϵ and of δ, but Re ϵ can be deduced from them. A direct measurement of Re ϵ is presented by equation (2.2) (Aharony, 1970): One has to measure the number of $\bar{K}^0$'s in a beam initiated by K^0 and determine the coefficient of equation (2.2). Such experiments have been discussed by Crawford (Crawford, 1965). In a time inverted contracting world this experiment will give a different sign for Re ϵ. Note, that we have yet no theoretical way to relate the sign of Re ϵ with the oscillation phase of the universe (Zweig, 1967). Still, if the definitions of matter and antimatter are agreed, the sign of Re ϵ is fixed by them since it is related to the difference between the fractions of K^0 and of $\bar{K}^0$ in K_L and K_S, and is thus determined by the experiments mentioned above. (These differences involve $\mathrm{Re}(\epsilon+\delta)$ and $\mathrm{Re}(\epsilon-\delta)$, and thus fix both Re ϵ and Re δ.)

Because of this ambiguity, it is interesting to consider a beam ending (or starting—in the time-inverted contracting world) at time $t=0$ as pure $\bar{K}^0$. Using the rule following equation (2.2) and equation (2.5) we find

$$P^{\bar{K}^0}(K^0,-t) = R^{K^0}(\bar{K}^0,t) \tag{3.1}$$

One might thus think that there is no distinction between the two oscillation phases if we invert the definitions of matter–antimatter. But this is not so, since in this case we shall have no symmetry between $P^{\bar{K}^0}(\bar{K}^0,t')$ and $P^{K^0}(K^0,t)$ unless $\delta=0$. Thus, only if *CPT* is conserved, one regains a complete symmetry if one defines $\bar{K}^0$ as K^0, and vice versa.

One might try to avoid the difference between the decay formulae in the expanding and in the time-inverted contracting phases, by assuming that in the time-inverted contracting universe one deals with different kinds of basis states $|K^{0\prime}\rangle$ and $|\bar{K}^{0\prime}\rangle$, instead of $|K^0\rangle$ and $|\bar{K}^0\rangle$, for which the physical behaviour is similar to equations (2.1)–(2.2). But it is easy to check that no combination of $|K^0\rangle$ and $|\bar{K}^0\rangle$ will give the same behaviour unless Re $\epsilon=0$.

4. *Conclusion*

If both *CPT* and *T* are not conserved, an experiment counting the fractions of K^0's and of $\bar{K}^0$'s in a K^0 beam will distinguish between the expanding and contracting phases of the universe oscillation, thus forming a relation between microscopic and cosmological arrows of time.

If *CPT* is conserved, this distinction may be resolved by inverting the definitions of matter–antimatter.

Even if *CPT* is conserved, there remains the question of the behaviour of a beam of K^0's beginning at $t=0$ and remaining—through the maximum expansion phase at $\tau/2$—until the end of a period, τ. Had it been possible to conserve such a beam, including information concerning its behaviour at short times, an asymmetry is due to appear at the time $\tau - t$.

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